Math. 640 Fall, 2018

Homework 3, due September 19

1–2. Exercises 10.1 and 10.5, pp. 58–59.

3. *[Milne p. 27]* Let $\mathcal{U}$ be the subspace of $\mathbf{R}^3$ consisting of the vectors $(0, y, z)$, and let $\mathcal{W}$ be the subspace spanned by $(2, 1, 0)$ and $(1, 2, 3)$. Which vectors are in $\mathcal{U} \cap \mathcal{W}$? Which vectors are in $\mathcal{U} + \mathcal{W}$? Is the sum direct?

4. *[cf. Milne p. 35]* Referring back to Exercise 2 of the last assignment: In those cases which *are* subspaces of $\mathbf{R}^3$, find a basis for a direct complement.

5. *[Milne p. 35]* Find the codimension of the following subspaces of the space of continuous functions $f(t)$ on $[0, 1]$:

   (a) subspace defined by conditions $f\left(\frac{1}{2}\right) = 0, \quad f\left(\frac{1}{3}\right) = f\left(\frac{2}{3}\right)$.

   (b) subspace defined by conditions

$$\int_0^1 \left(t - \frac{1}{2}\right)^n f(t)\, dt = 0, \qquad n = 0,\ 1,\ 2.$$

   Hint: Read about orthonormal bases and the Gram–Schmidt construction first.

6–10. Exercises 12.1, 12.2, 12.5, 12.6, 12.7, p. 69.

Note: Ex. 12.6 is misstated (how?).