Math. 640 Fall, 2018

Homework 4, due September 26

1–3. Exercises 13.3 *[for $\dim \mathcal{U} < \infty$]*, 13.5, 13.7, p. 74.

4. (a) Show that every formula of the form

$$\left(\underline{A}\vec{v}\right)^j \equiv \sum_k A^j{}_k v^k \qquad (k = 1, \ldots, n; \quad j = 1, \ldots, m)$$

defines a linear mapping $\underline{A}$ of $\mathcal{F}^n$ into $\mathcal{F}^m$ (where $\mathcal{F}$ is the field).

(b) Show that this correspondence between linear operators and $m \times n$ matrices is one-to-one (i.e., two different matrices can't represent the same operator (with respect to the same bases), nor vice versa).

5. Complete the proof of Theorem 15.8 by showing that $\dim \operatorname{ran} \underline{A} = \infty$ implies $\dim \operatorname{dom} \underline{A} = \infty$.

6. (a) Show that if $\omega = n\pi$ ($n \in \mathbf{Z}$, $n \neq 0$), then

$$f''(t) + \omega^2 f(t) = g(t) \tag{E}$$

has *many* solutions satisfying

$$f(0) = 0 = f(1), \tag{C$_1$}$$

if it has any such solutions at all.

(b) Show that if $\omega \neq n\pi$, then (E) has at most *one* solution satisfying (C_1).

7. Using methods from an elementary ODE course (variation of parameters),

(a) Show that if $\omega \neq n\pi$, then (E) has exactly one solution satisfying (C_1).

(b) Show that if $\omega = n\pi$, then

(i) If $\int_0^1 g(t) \sin n\pi t \, dt = 0$, then a solution exists satisfying (C_1) (not unique).

(ii) If this integral is not 0, then no solution exists satisfying (C_1).