Math. 640 Fall, 2018

Homework 5, due October 3

1. Complete the proof of Theorem 16.1 by showing that $\dim \mathcal{L}(\mathcal{V},\mathcal{U}) = \infty$ if either $\dim \mathcal{V}$ or $\dim \mathcal{U}$ equals ∞. (Try to adapt the proof on pp. 94–95.)

2. How is the matrix of $\underline{A}^*$ related to the matrix of $\underline{A}$ when the bases used are not orthonormal? Use the following notation:

$$\underline{A}\colon \mathcal{V} \to \mathcal{U}, \qquad \dim \mathcal{V} = n, \qquad \dim \mathcal{U} = m,$$

$$\vec{v} = \sum_{k=1}^{n} v^k \vec{b}_k \,, \qquad \vec{u} = \sum_{j=1}^{m} u^j \vec{d}_j \,,$$

$$\vec{b}_{k_1} \cdot \vec{b}_{k_2} \equiv g_{k_1 k_2} \,, \qquad \vec{d}_{j_1} \cdot \vec{d}_{j_2} \equiv h_{j_1 j_2} \,.$$

3. Exercise 16.3, p. 96.

4–6. Exercises 17.2, 17.3, 17.4, p. 104.

Hint for 17.2: Start by considering $\underline{B}$ = any projection with one-dimensional range.

7. Let $\underline{A} : \mathbf{C}^3 \to \mathbf{C}^2$ be defined by the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \end{pmatrix}$.

   (a) Discuss the solvability of the homogeneous equation $\underline{A}\vec{v} = 0$.

   (b) For what $\vec{b}$'s is the equation $\underline{A}\vec{v} = \vec{b}$ solvable? Is the solution unique? (Use the Fredholm method — i.e., look at the kernel of $\underline{A}^*$.)

8. Same as 7, with $\underline{A} : \mathbf{C}^3 \to \mathbf{C}^4$ defined by $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 1 & 3 & 1 \\ -1 & 0 & 0 \end{pmatrix}$.

9. Write out the Fredholm integral equation described in the paragraph "A MORE REALISTIC APPLICATION OF THIS IDEA" in the notes on Fredholm theory. Example 2. (Use the method of variation of parameters to construct the appropriate integral kernel, $K(t,s)$. You need not repeat details of steps you successfully executed on the previous assignment.)